

What quantum mechanics describes is discontinuous motion

Gao Shan

Institute of Quantum Mechanics

11-10, NO.10 Building, YueTan XiJie DongLi, XiCheng District

Beijing 100045, P.R.China

E-mail: gaoshan.iqm@263.net

We show that the natural motion of particles in continuous space-time (CSTM) is not classical continuous motion (CCM), but one kind of essentially discontinuous motion, the wave function in quantum mechanics is the very mathematical complex describing this kind of motion, and Schrödinger equation is just its simplest nonrelativistic motion equation, we call such motion quantum discontinuous motion or quantum motion; furthermore, we show that, when considering gravity the space-time will be essentially discrete, and the motion in discrete space-time (DSTM) will naturally result in the collapse process of the wave function, this finally brings about the appearance of classical continuous motion (CCM) in macroscopic world.

I. INTRODUCTION

The analysis about motion has never ceased since the old Greece times, but from Zeno Paradox to Einstein's relativity [6], only CCM is discussed, and its uniqueness is taken for granted ever since, but as to whether or not CCM is the only possible and objective motion, whether CCM is the real motion or apparent motion, no one has given a definite answer up to now, in fact, people have been indulging in the study of the motion law, but omitted the study of the motion itself.

On the other hand, we have entered into the microscopic world for nearly one century, but our understanding about it is still in confusion, the orthodox view [5] renounces CCM in microscopic world, but permits no existence of objective motion mode for the microscopic particles, while the opponents [3,4,7] still recourse to CCM to lessen the pain of losing realism, no other objective motion modes have been presented for the microscopic particles till now, thus the above problem is more urgent than ever.

In this paper, we will mainly address the above problem, after given a deep logical and physical analysis about motion, we demonstrate that the natural motion in continuous space-time or CSTM is not CCM, but one kind of essentially discontinuous motion, we call it quantum discontinuous motion or quantum motion, since we show that the wave function in quantum mechanics is the very mathematical complex describing it, and Schrödinger equation of the wave function is also its simplest nonrelativistic motion equation; Furthermore, since the combination of quantum mechanics and general relativity will result in the discreteness of space-time, namely the real space-time will be essentially discrete, we further study the motion in discrete space-time, or DSTM, and demonstrate that it will naturally result in the collapse process of the wave function, and finally bring about the appearance of CCM in macroscopic world.

The plan of this paper is as follows: In Sect. 2 we first give a general analysis about CSTM, the motion state of particle is physically defined, its general form and description are also given based on the mathematical analysis in the Appendix. In Sect. 3 we work out the simplest evolution law of CSTM, which turns out to be Schrödinger equation in quantum mechanics. In Sect. 4 we give a strict physical definition of CSTM, and further discuss the constant \hbar involved in its law. In Sect. 5 we point out that space-time is essentially discrete due to the ubiquitous existence of gravity, and give a simple demonstration. In Sect. 6 we further give a general analysis about DSTM, and the general form of motion state in such space-time is given. In Sect. 7 the evolution law of DSTM is worked out, and we demonstrate that it will naturally result in the collapse process of the wave function. In Sect. 8 we further show that CCM and its evolution law can be consistently derived from the evolution law of DSTM. At last, conclusions are given.

II. GENERAL ANALYSIS ABOUT MOTION IN CONTINUOUS SPACE-TIME (CSTM)

In this section, we will give a deep logical and physical analysis about CSTM.

A. The motion state of particle

First, we should define the motion state of particle, there are two alternatives, one is the instant state of particle, the other is the infinitesimal interval state of particle, it has been generally accepted that the motion state of particle should be the infinitesimal interval state of particle, not the instant state of particle, while people usually omit their essential difference, here we will present some of them.

(1).The instant state of particle contains only one point in space, its potential in mathematics is zero, while the infinitesimal interval state of particle contains infinite innumerable points in space, its potential in mathematics is ζ_1 .

(2).The instant state of particle contains no motion, but only the existence of particle, while the infinitesimal interval state of particle may contain abundant motion elements, since it contains infinite innumerable points in space.

(3).The instant state of particle possesses no physical meaning, since we can not access it through physical measurement, while the infinitesimal interval state of particle possesses real physical meaning, since we can measure it by means of the following infinite process: $\Delta t \rightarrow dt$.

(4).We can only find and confirm the law for the infinitesimal interval state of particle, while as to the instant state of particle, even if its law exists, we can not find it, let alone confirm it.

In fact, in physics there exist only the description quantities defined during infinitesimal time interval, this fact can be seen from the familiar differential quantities such as dt and dx , whereas the quantities defined at instants come only from mathematics, people always mix up these two kinds of quantities, this is a huge obstacle for the development of physics. Thus we can only discuss the motion state and relevant quantities defined during infinitesimal time interval, as well as their differential laws, if we study the point set corresponding to real physical motion.

For simplicity, in the following we say the motion state of particle at one instant, but it still denotes the infinitesimal interval state of particle, not the instant state of particle.

B. The general form of the motion state of particle

Secondly, we will give the general form of the motion state of particle, according to the analysis about point set (see Appendix), the natural assumption in logic is that the motion state of particle in infinitesimal time interval is a general dense point set in space, since we have no a priori reason to assume a special form, its proper description is the measure density $\rho(x, t)$ and measure density fluid $j(x, t)$.

Certainly, at some instant t the motion state of particle may assume some kind of special form, such as the continuous point set described by dx or $\rho(x, t) = \delta(x - x(t))$, but whether or not this kind of special form can exist for other instants should be determined by the motion law, not our prejudices.

III. THE EVOLUTION OF MOTION IN CONTINUOUS SPACE-TIME (CSTM)

In the following, we will give the main clues for finding the possible evolution equations of CSTM, and show that Schrödinger equation in quantum mechanics is just its simplest nonrelativistic evolution equations. Here we mainly analyze one-dimension motion, but the results can be easily extended to three-dimension situation.

A. The first motion principle

First, we should find the first motion principle similar to the first Newton principle, this means that we need to find the simplest solution of the motion equation, in which we can get the invariant quantity during free motion, it is evident that the simplest solution of the motion equation is:

$$\frac{\partial \rho(x, t)}{\partial t} = 0 \quad (1)$$

$$\frac{\partial j(x, t)}{\partial x} = 0 \quad (2)$$

$$\frac{\partial j(x, t)}{\partial t} = 0 \quad (3)$$

$$\frac{\partial \rho(x, t)}{\partial x} = 0 \quad (4)$$

using the relation $j(x, t) = \rho(x, t) \cdot v$ we can further get the solution, namely $\rho(x, t) = 1, j(x, t) = v = p/m$, where m is the mass of the particle, p is defined as the momentum of particle. Now, we get the first motion principle, namely during the free motion of particle, the momentum of the particle is invariant, but it can be easily seen that, contrary to classical continuous motion, for the free particle with one constant momentum, its position will not be limited in the infinitesimal space dx , but spread throughout the whole space with the same position measure density.

Similar to the quantity position, the natural assumption in logic is also that the momentum (motion) state of particle in infinitesimal time interval is still a general dense point set in momentum space, thus we can also define the momentum measure density $f(p, t)$, and the momentum measure fluid density $J(p, t)$, their meanings are similar to those of position.

B. Two kinds of description bases

Now, we have two description quantities, one is position, the other is momentum, and position descriptions $\rho(x, t)$ and $j(x, t)$ provide a complete local description of the motion state, we may call it local description basis, similarly momentum descriptions $f(p, t)$ and $J(p, t)$ provide a complete nonlocal description of the motion state, since for the particle with any constant momentum, its position will spread throughout the whole space with the same position measure density, we may call it nonlocal description basis.

Furthermore, at any instant the motion state of particle is unique, thus there should exist a one-to-one relation between these two kinds of description bases, namely there should exist a one-to-one relation between position description (ρ, j) and momentum description (f, J) , and this relation is irrelevant to the concrete motion state, in the following we will mainly discuss how to find this one-to-one relation, and our analysis will also show that this relation essentially determines the evolution of motion.

C. One-to-one relation

First, it is evident that there exists no direct one-to-one relation between the measure density functions $\rho(x, t)$ and $f(p, t)$, since even for the above simplest situation, we have $\rho(x, t) = 1$ and $f(p, t) = \delta^2(p - p_0)^*$, and there is no one-to-one relation between them.

Then in order to obtain the one-to-one relation, we have to create new properties on the basis of the above position description (ρ, j) and momentum description (f, J) , this needs a little mathematical trick, here we only give the main clues and the detailed mathematical demonstrations are omitted, first, we disregard the time variable t and let $t = 0$, as to the above free evolution state with one momentum, we have $(\rho, j) = (1, p_0/m)$ and $(f, J) = (\delta^2(p - p_0), 0)$, thus we need to create a new position state function $\psi(x, 0)$ using 1 and p_0/m , a new momentum state function $\varphi(p, 0)$ using $\delta^2(p - p_0)$ and 0, and find the one-to-one relation between these two state functions, this means there exists a one-to-one transformation between the state functions $\psi(x, 0)$ and $\varphi(p, 0)$, we generally write it as follows:

$$\psi(x, 0) = \int_{-\infty}^{+\infty} \varphi(p, 0) T(p, x) dp \quad (5)$$

where $T(p, x)$ is the transformation function and generally continuous and finite for finite p and x , since the function $\varphi(p, 0)$ will contain some form of the basic element $\delta^2(p - p_0)$, normally we may expand it as $\varphi(p, 0) = \sum_{i=1}^{\infty} a_i \delta^i(p - p_0)$, while the function $\psi(x, 0)$ will contain the momentum p_0 , and be generally continuous and finite for finite x , then it is evident that the function $\varphi(p, 0)$ can only contain the term $\delta(p - p_0)$, because the other terms will result in infiniteness.

On the other hand, since the result $\varphi(p, 0) = \delta(p - p_0)$ implies that there exists the simple relation $f(p, 0) = \varphi(p, 0)^* \varphi(p, 0)^\dagger$, and owing to the equality between the position description and momentum description, we also have the similar relation $\rho(x, 0) = \psi(x, 0)^* \psi(x, 0)$, thus we may let $\psi(x, 0) = e^{iG(p_0, x)}$ and have $T(p, x) = e^{iG(p, x)}$, then

* This result can be directly obtained when considering the general normalization relation $\int_{\Omega} \rho(x, t) dx = \int_{\Omega} f(p, t) dp$.

† Evidently, another simple relation $f(p, 0) = \varphi(p, 0)^2$ permit no existence of one-to-one relation

considering the symmetry between the properties position and momentum[‡], we have the general extension $G(p, x) = \sum_{i=1}^{\infty} b_i(px)^i$, furthermore, this kind of symmetry also results in the symmetry between the transformation $T(p, x)$ and its reverse transformation $T^{-1}(p, x)$, where $T^{-1}(p, x)$ satisfies the relation $\varphi(p, 0) = \int_{-\infty}^{+\infty} \psi(x, 0)T^{-1}(p, x)dx$, thus we can only have the term px in the function $G(p, x)$, and the resulting symmetry relation between these two transformations will be $T^{-1}(p, x) = T^*(p, x) = e^{-ipx}$, we let $b_1 = 1/\hbar$, where \hbar is a constant quantity (for simplicity we let $\hbar = 1$ in the following discussions), then we get the basic one-to-one relation, it is $\psi(x, 0) = \int_{-\infty}^{+\infty} \varphi(p, 0)e^{ipx}dp$, where $\psi(x, 0) = e^{-ip_0x}$ and $\varphi(p, 0) = \delta(p - p_0)$, it mainly results from the essential symmetry involved in CSTM itself.

In order to further find how the time variable t is included in the functions $\psi(x, t)$ and $\varphi(p, t)$, we may consider the superposition of two single momentum states, namely $\psi(x, t) = \frac{1}{\sqrt{2}}[e^{ip_1x - ic_1(t)} + e^{ip_2x - ic_2(t)}]$, then the position measure density is $\rho(x, t) = [1 + \cos(\Delta c(t) - \Delta px)]/2$, where $\Delta c(t) = c_2(t) - c_1(t)$ and $\Delta p = p_2 - p_1$, now we let $\Delta p \rightarrow 0$, then we have $\rho(x, t) \rightarrow 1$ and $\Delta c(t) \rightarrow 0$, especially using the measure conservation relation we can get $dc(t)/dt = dp \cdot p/m$, namely $dc(t) = d(p^2/m) \cdot t$ or $dc(t) = dE \cdot t$, where $E = p^2/m$, is defined as the energy of the particle in the nonrelativistic domain, thus as to any single momentum state we have the time-included formula $\psi(x, t) = e^{ipx - iEt}$.

In fact, there may exist other complex forms for the state functions $\psi(x, t)$ and $\varphi(p, t)$, for example, they are not the above simple number functions but multidimensional vector functions such as $\psi(x, t) = (\psi_1(x, t), \psi_2(x, t), \dots, \psi_n(x, t))$ and $\varphi(p, t) = (\varphi_1(p, t), \varphi_2(p, t), \dots, \varphi_n(p, t))$, but the above one-to-one relation still exists for every component function, and these vector functions still satisfy the above modulo square relations, namely $\rho(x, t) = \sum_{i=1}^n \psi_i(x, t)^* \psi_i(x, t)$ and $f(p, t) = \sum_{i=1}^n \varphi_i(p, t)^* \varphi_i(p, t)$, these complex forms will correspond to the particles with more complex structure, say, involving more inner properties of the particle such as charge and spin etc.

At last, since the one-to-one relation between the position description and momentum description is irrelevant to the concrete motion state, the above one-to-one relation for the free motion state with one momentum should hold true for any motion state, and the states satisfying the one-to-one relation will be the possible motion states. Furthermore, it is evident that this one-to-one relation will directly result in the famous Heisenberg uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$.

D. The evolution law of motion in continuous space-time (CSTM)

Now, we will work out the evolution law of CSTM.

First, as to the free motion state with one momentum, namely the single momentum state $\psi(x, t) = e^{ipx - iEt}$, using the above definition of energy $E = \frac{p^2}{m}$ and including the constant quantity \hbar we can easily find its nonrelativistic evolution law, which is

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x, t)}{\partial x^2} \quad (6)$$

then owing to the linearity of this equation, this evolution equation also applies to the linear superposition of the single momentum states, namely all possible free motion states, or we can say, it is the free evolution law of CSTM.

Secondly, we will consider the evolution law of CSTM under outside potential, when the potential $U(x, t)$ is a constant U , the evolution equation will be

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x, t)}{\partial x^2} + U \cdot \psi(x, t) \quad (7)$$

then when the potential $U(x, t)$ is related to x and t , the above form will still hold true, namely

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x, t)}{\partial x^2} + U(x, t) \cdot \psi(x, t) \quad (8)$$

for three-dimension situation the equation will be

$$i\hbar \frac{\partial \psi(\mathbf{x}, \mathbf{t})}{\partial t} = -\frac{\hbar^2}{2m} \cdot \nabla^2 \psi(\mathbf{x}, \mathbf{t}) + \mathbf{U}(\mathbf{x}, \mathbf{t}) \cdot \psi(\mathbf{x}, \mathbf{t}) \quad (9)$$

[‡]This symmetry essentially stems from the equivalence between these two kinds of descriptions, the direct implication is for $\rho(x, 0) = \delta^2(x - x_0)$ we also have $f(p, 0) = 1$.

this is just the Schrödinger equation in quantum mechanics[§].

At last, the above analysis also shows that the state function $\psi(x, t)$ provides a complete description of CSTM, since CSTM is completely described by the measure density $\rho(x, t)$ and measure fluid density $j(x, t)$, and according to the above evolution equation the state function $\psi(x, t)$ can be expressed by these two functions, namely $\psi(x, t) = \rho^{1/2} \cdot e^{iS(x, t)/\hbar}$, where $S(x, t) = m \int_{-\infty}^x j(x', t)/\rho(x', t) dx' + C(t)$ ^{**}, and these two functions can also be expressed by the state function, namely $\rho(x, t) = |\psi(x, t)|^2$ and $j(x, t) = [\psi^* \partial \psi / \partial t - \psi \partial \psi^* / \partial t] / 2i$, then there exists a one-to-one relation between $(\rho(x, t), j(x, t))$ and $\psi(x, t)$ when omitting the absolute phase, thus the state function $\psi(x, t)$ also provides a complete description of CSTM.

On the other hand, we can see that the absolute phase of the wave function $\psi(x, t)$, which may depend on time, is useless for describing CSTM, since according to the above analysis it disappears in the measure density $\rho(x, t)$ and measure fluid density $j(x, t)$, which completely describe CSTM, thus it is natural that the absolute phase of the wave function possesses no physical meaning.

IV. FURTHER DISCUSSIONS ABOUT MOTION IN CONTINUOUS SPACE-TIME (CSTM)

A. The definition of motion in continuous space-time (CSTM)

Now we can give the physical definition of CSTM in three-dimension space, the definition for other abstract spaces or many-particle situation can be easily extended.

- (1). The motion of particle in space is described by dense point set in four-dimension space and time.
- (2). The motion state of particle in space is described by the position measure density $\rho(\mathbf{x}, \mathbf{t})$ and position measure fluid density $\mathbf{j}(\mathbf{x}, \mathbf{t})$ of the corresponding dense point set.
- (3). The evolution of motion corresponds to the evolution of the dense point set, and the simplest evolution equation is Schrödinger equation in quantum mechanics.

Compared with classical continuous motion, we may call CSTM quantum discontinuous motion, or quantum motion, the commonness of these two kinds of motion is that they are both the motion of particle, namely the moving object exists only in one position in space at one instant, their difference lies in the moving behavior, namely the behavior of the particle during infinitesimal time interval $[t, t+dt]$, for classical motion, the particle is limited in a certain local space interval $[V, V+dV]$, while for quantum motion, the particle moves throughout the whole space with a certain position measure density $\rho(\mathbf{x}, \mathbf{t})$.

In fact, all physical states of CSTM are defined during infinitesimal time interval in the meaning of measure, not at one instant, for example, the single momentum state $\psi_p(x, t) = e^{ipx - iEt}$, especially even the single position state $\psi(x, t) = \delta(x - x_0)$ is still defined during infinitesimal time interval.

B. Some discussions about the constant \hbar

First, from the above analysis about CSTM, we can understand why the constant \hbar with dimension $J \cdot s$ should appear in the evolution equation of CSTM, or Schrödinger equation, the existence of \hbar essentially results from the equivalence between the nonlocal momentum description and local position description of CSTM, it is this equivalence that results in the one-to-one relation between these two kinds of descriptions, which requires the existence of a certain constant \hbar with dimension $J \cdot s$ to cancel out the dimension of the physical quantities px and Et in the relation, at the same time, the existence of \hbar also indicates some kind of balance between the dispersion of the position distribution and that of momentum distribution limited by the one-to-one relation (there is no such limitation for CCM and $\hbar = 0$), or we can say, the existence of \hbar essentially indicates some kind of balance between the nonlocality and locality of motion in continuous space-time.

[§]In fact, Schrödinger equation is also the simplest evolution equation of dense point set, this fact manifests the identity of physical reality and mathematical reality to some extent.

^{**}When in three-dimension space, the formula for $S(\mathbf{x}, \mathbf{y}, \mathbf{z}, t)$ will be $S(x, y, z, t) = m \int_{-\infty}^x j_x(x', y, z, t)/\rho(x', y, z, t) dx' + C(t) = m \int_{-\infty}^y j_y(x, y', z, t)/\rho(x, y', z, t) dy' + C(t) = m \int_{-\infty}^z j_z(x, y, z', t)/\rho(x, y, z', t) dz' + C(t)$, since in general there exists the relation $\nabla \times \{\mathbf{j}(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t})/\rho(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t})\} = \mathbf{0}$.

Secondly, even though the appearance of \hbar in the evolution equation of CSTM is inevitable, its value can not be determined by CSTM itself, we only know that \hbar possesses a finite nonzero value, certainly, just like the other physical constants such as c and G , its value can be determined by the experience, but there may exist some deeper reasons for the special value of \hbar in our universe, although motion can not determine this value alone, the solution may have to resort to other subtle realities in this world, for example, gravity (G), space-time(c), or even the existence of our mankind.

V. THE REAL SPACE-TIME IS DISCRETE

The combination of quantum mechanics and general relativity strongly implied space-time is essentially discrete, and the minimum space-time unit will be Planck size T_p and L_p , thus owing to the ubiquitous existence of gravity, the real space-time will be essentially discrete with the minimum size T_p and L_p .

Here we will give a simple operational demonstration about the discreteness of space-time, consider a measurement of the length between points A and B, at point A place a clock with mass m and size a to register time, at point B place a reflection mirror, when $t=0$ a photon signal is sent from A to B, at point B it is reflected by the mirror and returns to point A, then the clock registers the return time, for classical situation the measured length will be $L = \frac{1}{2}ct$, but when considering quantum mechanics and general relativity, the existence of the clock introduces two kinds of uncertainties to the measured length, the uncertainty resulting from quantum mechanics is: $\delta L_{QM} \geq (\frac{\hbar L}{mc})^{1/2}$, the uncertainty resulting from general relativity is: $\delta L_{GR} \cong \frac{1}{2} \frac{2Gm}{c^2} \ln \frac{a+L}{a} \geq \frac{Gm}{c^2}$, then the total uncertainties is: $\delta L = \delta L_{QM} + \delta L_{GR} \geq (L \cdot L_p^2)^{1/3}$, where $L_p = (\frac{G\hbar}{c^3})^{1/2}$, is Planck length, thus we conclude that the minimum measurable length is Planck length L_p , in a similar way, we can also work out the minimum measurable time, it is just Planck time T_p .

VI. GENERAL ANALYSIS ABOUT MOTION IN DISCRETE SPACE-TIME (DSTM)

In the discrete space-time, there exist absolute minimum sizes T_p and L_p , namely the minimum distinguishable size of time and position of the particle is respectively T_p and L_p , thus in physics the existence of the particle is no longer in one position at one instant as in the continuous space-time, but limited in a space interval L_p during a finite time interval T_p , it can be seen that this state corresponds to the instant state of particle in continuous space-time, we define it as the instant state of particle in discrete space-time, this state evidently contains no motion, but only the existence of particle.

Furthermore, during the finite time interval T_p the particle can only be limited in a space interval L_p , since if it can move throughout at least two different local regions with separation size larger than L_p during the time interval T_p , then there essentially exists a smaller distinguishable finite time interval than T_p , which evidently contradicts the fact that T_p is the minimum time unit, thus the discreteness of space-time essentially results in the existence of local position state of the particle, in which the particle stays in a local region with size L_p for a time interval T_p , we may call such general local position state Planck cell state.

Similar to the analysis of the motion state in continuous space-time, in discrete space-time, the natural assumption in logic is that the motion state of particle in finite time interval, which is much longer than T_p but still small enough, is a general discrete point set or cell set in space, since we have no a priori reason to assume a special form, its proper description is still the measure density $\rho(x, t)$ and measure density fluid $j(x, t)$, but in the meaning of time average.

Certainly, we can also get the motion state of particle in discrete space-time from that in continuous space-time, since in continuous space-time the particle, which instant state is the particle being in one position at one instant, moves throughout the whole space during infinitesimal time interval, while in discrete space-time the instant state of particle turns to be the particle being in a space interval L_p during a finite time interval T_p , the motion state of particle in discrete space-time will naturally be that during a finite time interval much larger than T_p the particle moves throughout the whole space with the position measure density $\rho(x, t)$ in the meaning of time average.

Now the visual physical picture of DSTM will be that during a finite time interval T_p the particle will stay in a local region with size L_p , then it will still stay there or "jump" to another local region, which may be very far from the original region, while during a time interval much larger than T_p the particle will move throughout the whole space with a certain average position measure density $\rho(x, t)$.

As we can see, on the one hand, the particle undergoing DSTM stays in a local region during infinitesimal time interval, this may generate the display of CCM, on the other hand, during a finite time interval much larger than T_p the particle will continually jumps from one local region to another local region, and move throughout the whole space with a certain position measure density $\rho(x, t)$, this may generate the display of CSTM, then DSTM is evidently some

kind of unification of CCM and CSTM, in fact, owing to the ubiquitous existence of quantum and gravity, space-time is essentially discrete, and DSTM will be the only real motion in Nature, while CSTM and CCM are the apparent motion modes, they are only two ideal approximators of DSTM in microscopic and macroscopic world, thus DSTM is just the lost reality unifying quantum motion (CSTM) and classical motion (CCM), and it will undoubtedly provide an uniform realistic picture for microscopic world and macroscopic world, the following analysis will confirm this conclusion more convincingly.

VII. THE EVOLUTION OF MOTION IN DISCRETE SPACE-TIME (DSTM)

A. A general discussion

Since CSTM is some kind of time average of DSTM, the evolution of DSTM will follow the evolution law of CSTM in the meaning of time average, on the other hand, the particle undergoing DSTM does stay in a local region for a finite nonzero time interval, and jump from this local region to another local region stochastically, thus the position measure density $\rho(x, t)$ of the particle will be essentially changed in a stochastic way due to the finite nonzero stay time in different stochastic region^{††}, and the corresponding wave function will be also stochastically changed, then this kind of stochastic jump inevitably introduce the stochastic element to the evolution, so the evolution law of DSTM will be the combination of the deterministic linear evolution and stochastic nonlinear evolution, in the following we will work out this law.

B. Two rules

At first, since CSTM is some kind of average of DSTM during a finite time interval much larger than T_p , thus the position measure density $\rho(x, t)$ of the particle undergoing CSTM will be also the average of the position distribution of the particle undergoing DSTM during this time interval, then it is natural that the position of the particle undergoing DSTM will satisfy the position measure density $\rho(x, t)$, namely for DSTM the stochastic stay position of the particle satisfies the distribution

$$P(x, t) = |\psi(x, t)|^2 \quad (10)$$

this is the first useful rule for finding the evolution law of DSTM.

Secondly, according to the definition of the position measure density $\rho(x, t)$, the finite nonzero stay time of the particle in a local region evidently implies that the position measure density $\rho(x, t)$ in that region will be increased after this finite nonzero stay time interval, and the increase will be larger when the stay time is longer. We consider the general situation that the particle undergoing DSTM stays in a local region L_p for a time interval T , in the first rank approximation the increase of the position measure density $\rho(x, t)$ in this region can be written as follows after normalization:

$$\rho(x, t + T) = \frac{1}{A(T)}(\rho(x, t) + T/T_m) \quad (11)$$

where $A(T)$ is the normalization factor, T_m is a certain time size to be determined, which may be relevant to the concrete motion state of the particle, this will be the second useful rule.

We first work out the normalization factor $A(t)$, considering the following two conditions:(1)when $T = 0, \rho(x, t+T) = \rho(x, t)$, and $A(0) = 1$;(2)when $T \rightarrow \infty, \rho(x, t + T) \rightarrow 1$, and $A(\infty) \rightarrow T/T_m$, we can get $A(T) = 1 + T/T_m$, then the above formula will be:

$$\rho(x, t + T) = \frac{\rho(x, t) + T/T_m}{1 + T/T_m} \quad (12)$$

or it can be written as follows:

^{††}For CSTM, the stay time of the particle in any position is zero, so its position measure density $\rho(x, t)$ is not influenced by the stochastic jump.

$$\Delta\rho(x, t) = \frac{T}{T_m + T}(1 - \rho) \quad (13)$$

in general, when $T > T_p$, namely when the particle undergoing DSTM stays in a local region L_p for a time interval longer than T_p , we can divide the whole time interval T into many Planck cell T_p , and the above formula is still valid for every cell, thus in the following discussions we let $T = T_p$ for simplicity.

In order to further find the formula of T_m , we need to study the limitation on the jump of the particle undergoing DSTM, since the item T_m just denotes this kind of limitation, this can be seen from the following two extreme situations: (1) when $T_m \rightarrow \infty$, we have $\Delta\rho \rightarrow 0$, this denotes that the position measure density will be not influenced by the jump, and the particle can jump freely; (2) $T_m \rightarrow 0$, we have $\rho \rightarrow 1$, this denotes that the position measure density will turn to be one in the region where the particle stays, and the position measure density in other regions will turn to be zero, so the particle can not jump at all. In fact, from the physical analysis about the jump we can see that the limitation results from the principle of energy conservation, according to which during a finite nonzero time interval Δt the possible change of energy ΔE_j resulting from jump will be limited by the uncertainty relation $\Delta E_j \approx \hbar/\Delta t$, now we consider two situations, first, if the total difference of energy ΔE between the original stay region and other regions satisfies the condition $\Delta E \gg \Delta E_j$, then the particle can hardly jump from its original region to other regions, namely after the stay time Δt the position measure density $\rho(x, t)$ in the original region will be greatly increased, especially when $\Delta E \rightarrow \infty$, we have $\rho(x, t) \rightarrow 1$, and $T_m \rightarrow 0^{\dagger\dagger}$; Secondly, if the total difference of energy ΔE between the original stay region and other regions satisfies the condition $\Delta E \ll \Delta E_j$, then the particle can jump more easily from its original region to other regions, namely after the stay time Δt the position measure density $\rho(x, t)$ will be only changed slightly, especially when $\Delta E \rightarrow 0$, we have $\Delta\rho(x, t) \rightarrow 0$, and $T_m \rightarrow \infty$. Then we can see that T_m is inversely proportional to ΔE , considering the dimension requirement their relation will be $T_m = \hbar/k\Delta E$, where k is a dimensionless constant.

Now, the change of the position measure density after stay time T_p can be formulated in a more complete way:

$$\rho(x, t + T) = \frac{\rho(x, t) + k\Delta E/E_p}{1 + k\Delta E/E_p} \quad (14)$$

or it can be written as follows:

$$\Delta\rho(x, t) = \frac{\Delta E}{kE_p + \Delta E}(1 - \rho) \quad (15)$$

where $E_p = \hbar/T_p$ is Planck energy, thus we get the second useful rule for finding the evolution law of DSTM.

C. The evolution law of motion in discrete space-time (DSTM)

Now, according to the above two rules, we can give the evolution equation of DSTM.

For simplicity but lose no generality, we consider a one-dimension initial wave function $\psi(x, 0)$, according to the above analysis, the concrete evolution equation of DSTM will be essentially one kind of revised stochastic evolution equation based on Schrödinger equation, here we assume the form of stochastic differential equation (SDE), it can be written as follows:

$$d\psi(x, t) = \frac{1}{i\hbar} H_Q \psi(x, t) dt + \frac{1}{2} \left[\frac{\delta_{xx_N}}{\rho(x, t)} - 1 \right] \frac{\Delta E(x_N, \overline{x_N})}{kE_p + \Delta E(x_N, \overline{x_N})} \psi(x, t) dt \quad (16)$$

where the first term in right side represents the evolution element resulting from CTSM, the average behavior of DSTM, H_Q is the corresponding Hamiltonian, the second term in right side represents the evolution element resulting from the stochastic jump resulting from DSTM itself, δ_{xx_N} is the discrete δ -function, k is a dimensionless constant, $\rho(x, t) = |\psi(x, t)|^2$, is the position measure density, $\Delta E(x_N, \overline{x_N})$ is the total difference of energy of the particle between the cell containing x_N and all other cells $\overline{x_N}$, x_N is a stochastic position variable, whose distribution is $P(x_N, t) = \rho(x_N, t)$.

In physics, this stochastic differential equation is essentially a discrete evolution equation, all the quantities are defined relative to the Planck cells T_p and L_p , and the equation should be also solved in a discrete way.

^{††}In fact, in this situation the wave function has collapsed into this local region in order to satisfy the requirement of energy conservation, and this also indicates that in order to satisfy the principle of energy conservation DSTM will naturally result in the collapse of the wave function.

D. Some further discussions

Now we will give some physical analyses about the above evolution equation of DSTM, first, the linear item in the equation will result in the spreading process of the wave function as for the evolution of CSTM, while the nonlinear stochastic item in the equation will result in the localizing process of the particle or collapse process of the wave function, this can also be seen qualitatively, since according to the nonlinear stochastic item, in the region where the position measure density is larger the stay time of the particle will be longer, moreover, the longer stay time of the particle in one region will further increase the position measure density in that region much more, thus this process is evidently one kind of positive feedback process, the particle will finally stay in a local region, and the wave function of particle will also collapse to that region, so the evolution of DSTM will be some kind of combination of the spreading process and localizing process.

Secondly, the strength of the spreading process and localizing process is mainly determined by the energy difference between different branches of the wave function, if the energy difference is so small, then the evolution of DSTM will be mainly dominated by the spreading process, or we can say, the display of DSTM will be more like that of quantum motion (CSTM), this is just what happens in microscopic world; while if the energy difference is so large, then the evolution of DSTM will be mainly dominated by the localizing process, or we can say, the display of DSTM will be more like that of classical motion (CCM), this is just what happens in macroscopic world, and the boundary of these two worlds can also be estimated, the following example indicates that the energy difference in the boundary may assume $\Delta E \cong \sqrt{\hbar E_p} \approx 7 \text{ Mev}$, the corresponding collapse time will be in the level of seconds.

Thirdly, if the particle finally stay in a local region during the evolution of DSTM, the localizing probability of the particle, or the collapse probability the wave function in one local region is just the initial position measure density of the particle in that region, namely the probability satisfies the Born rule in quantum mechanics, since the stochastic evolution of DSTM satisfies the Martingale condition, this can be seen from the following fact, namely during every jump the position measure density ρ satisfies the equation $P(\rho) = \rho P(\rho + \frac{\Delta E}{\hbar E_p + \Delta E}(1 - \rho)) + (1 - \rho)P(\rho - \frac{\Delta E}{\hbar E_p + \Delta E}\rho)$ [12], where $P(\rho)$ is the probability of ρ turning into one in one local region, namely the probability of the particle localizing in a local region, moreover, the solution of this equation is $P(\rho) = \rho$, this just means that the localizing probability of the particle in one region is just the initial position measure density of the particle in that region.

Fourthly, the collapse process resulting from the evolution of DSTM has no tails, since the evolution is essentially discrete, the wave function is just the description of the motion of particle, and its existence is only in the meaning of time average, while the particle, the real object, always exists in one local position, thus in the last stage of the collapse process, when the particle stays in one of the branches long enough it will de facto collapse into that branch owing to the limitation of energy conservation, and the wave function, the apparent "object", will also completely disappears in other branches^{§§}.

At last, the existence of DSTM will help to tackle the well-known time problem involved in formulating a complete theory of quantum gravity [13], since as to DSTM, the local position state of particle will be the only proper state, and the only real physical existence, during a finite time interval T_p the particle can only be limited in a local space interval L_p , namely there does not exist any essential superposition of different positions at all, the superposition of the wave function is only in the meaning of time average, thus the essential inconsistency of the superposition of different space-time in the theory of quantum gravity, which results from the existence of the essential superposition of the wave function, will naturally disappear, and the real physical picture based on DSTM will be that at any instant (during a finite time interval T_p) the structure of space-time determined by the existence of the particle (in a local space interval L_p) is definite or "classical", while during a finite time interval much larger than T_p but still small enough it will be stochastically disturbed by the stochastic jump of the particle undergoing DSTM, this kind of stochastic disturbance will be the real quantum nature of the space-time and matter.

E. One simple example

In this section, as one example we will analyze the DSTM evolution of a simple two-state system, and quantificationally show that the evolution of DSTM will indeed result in the collapse process of the wave function.

^{§§}If the wave function is taken as some kind of essential existence, and its evolution is essentially continuous, then the tails problem will be inevitable.

We suppose the initial wave function of the particle is $\psi(x, 0) = \alpha(0)^{1/2}\psi_1(x) + \beta(0)^{1/2}\psi_2(x)$, which is a superposition of two static states with different energy levels E_1 and E_2 , these two static states are located in separate regions R_1 and R_2 with the same size.

Since the energy of the particle inside the region of each static state is the same, we can consider the spreading space of both static states as a whole local region, and only study the stochastic jump between these two regions resulting from the evolution of DSTM, namely we directly consider the difference of the energy $\Delta E = E_2 - E_1$ between these two states, through some mathematical calculations we can work out the density matrix of the two-state system, it is:

$$\rho_{11}(t) = \alpha(0) \quad (17)$$

$$\rho_{22}(t) = \beta(0) \quad (18)$$

$$\rho_{12}(t) = [1 - (\frac{\Delta E}{kE_p + \Delta E})^2]^{t/2T_p} \sqrt{\alpha(0)\beta(0)} \approx (1 - \frac{(\Delta E)^2}{2k^2\hbar E_p}t) \sqrt{\alpha(0)\beta(0)} \quad (19)$$

$$\rho_{21}(t) = [1 - (\frac{\Delta E}{kE_p + \Delta E})^2]^{t/2T_p} \sqrt{\alpha(0)\beta(0)} \approx (1 - \frac{(\Delta E)^2}{2k^2\hbar E_p}t) \sqrt{\alpha(0)\beta(0)} \quad (20)$$

It is evident that these results confirm the above qualitative analysis definitely, namely, the evolution of DSTM indeed results in the collapse of the wave function describing DSTM, and the distribution of the collapse results satisfies the Born rule in quantum mechanics, besides, we also get the concrete collapse time for two-state system, it is $\tau_c \approx 2k^2 \frac{\hbar E_p}{(\Delta E)^2}$ ***.

VIII. THE APPEARANCE OF CLASSICAL MOTION IN MACROSCOPIC WORLD

The above analysis has indicated that, when the energy difference between different branches of the wave function is large enough, say for the macroscopic situation^{†††}, the linear spreading of the wave function will be greatly suppressed, and the evolution of the wave function will be dominated by the localizing process, in fact, the motion state of the particle will be only local position state in appearance, and the evolution of this state will be only still or continuously move in space, this is just the display of CCM in macroscopic world.

Furthermore, we will show that the evolution law of CCM can also be derived, in fact, some people have strictly given the demonstration based on revised quantum dynamics [9,10], here we simply use the Enrenfest theorem, namely $\frac{d\langle x \rangle}{dt} = \langle p \rangle$ and $\frac{d\langle p \rangle}{dt} = -\langle \frac{\partial U}{\partial x} \rangle$, as we have demonstrated, for macroscopic object its wave function will no longer spread, thus the average items in the theorem will represent the effective description quantities for the classical motion of the macroscopic object, and the classical motion law is also naturally derived in such a way, the result is $\frac{dx}{dt} = p$, the definition of the momentum, and $\frac{dp}{dt} = -\frac{\partial U}{\partial x}$, the motion equation.

IX. CONCLUSIONS

In this paper, we strictly demonstrate the logical inevitability of the existent form and evolution law of CSTM, the existence of discrete space-time in Nature and resulting real existence of DSTM and its evolution law, this not only explains the appearance of classical motion in macroscopic world, as well as quantum motion in microscopic world consistently and objectively, but also presents a clear logical connection between quantum motion and classical motion, and unveils the unified realistic picture of microscopic and macroscopic world.

***This result has also been obtained by Hughston [11] and Fivel [8] from different point of views, and discussed by Adler et al [1,2].

†††The largeness of the energy difference for macroscopic object results mainly from the environmental influences such as thermal energy fluctuations.

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Appendix:Mathematical Analysis About Motion In Continuous Space-time

First, we will give three general presuppositions about the relation between physical motion and mathematical point set, they are basic conceptions and correspondence rules before we discuss the physical motion of particles in continuous space-time.

- (1). Time and space in which the particle moves are both continuous point set.
- (2). The moving particle is represented by one point in time and space.
- (3). The motion of particle is represented by the point set in time and space.

The first presupposition defines the continuity of space-time, the second one defines the existent form of particle in time and space, the last one relates the physical motion of particle with the mathematical point set.

For simplicity but lose no generality, in the following we will mainly analyze the point set in two-dimension space-time, which corresponds to one-dimension motion in continuous space-time.

A. Point set and its law—a general discussion

As we know, the point set theory has been deeply studied since the beginning of this century, nowadays we can grasp it more easily, according to this theory, we know that the general point set is dense point set, whose basic property is the measure of the point set, while the continuous point set is one kind of special dense point set, its basic property is the length of the point set.

Naturally, as to the point set in two-dimension space-time, the general situation is the dense point set in this two-dimension space-time, while the continuous curve is one kind of extremely special dense point set, surely it is a wonder that so many points bind together to form one continuous curve by order, in fact, the probability for its natural formation is zero.

Now, we will generally analyze the law of the point set, as we know, the law about the points in point set, which can be called point law, is the most familiar law, and it is widely taken as the only rational law, for example, as to the continuous curve in two-dimension space-time there may exist a certain expressible analytical formula for the points in this special point set^{†††}, but as to the general dense point set in two-dimension space-time the point law possesses no mathematical meaning, since the dense point set is discontinuous everywhere, even if the difference of time is very small, or infinitesimal, the difference of space can be very large, then infinitesimal error in time will result in finite error in space, thus even if the point law exists we can not formulate it in mathematics, and owing to finite error in time determination and calculation, we can not prove it either, let alone predict the evolution of the point set using it, in one word, there does not exist point law for dense point set in mathematics.

B. Deep analysis about dense point set

Now, we will further study the differential description of point set in detail.

First, in order to find the differential description of the special dense point set—continuous curve, we may measure the rise or fall size of the space Δx corresponding to any finite time interval Δt near each instant t_j , then at any instant t_j we can get the approximate information about the continuous curve through the quantities Δt and Δx at that instant, and when the time interval Δt turns smaller, we will get more accurate information about the curve. In theory, we can get the complete information through this infinite process, that is to say, we can build up the

^{†††}People cherish this kind of point laws owing to their infrequent existence, but perhaps Nature detests and rejects them, since the probability of creating them is zero.

basic description quantities for the special dense point set—continuous curve, which are the differential quantities dt and dx , then given the initial condition the relation between dt and dx at all instants will completely describe the continuous curve.

Then, we will analyze the differential description of the general dense point set, as to this kind of point set, we still need to study the concrete situation of the point set corresponding to finite time interval near every instant. Now, when time is during the interval Δt near instant t_j , the points in space are no longer limited in the local space interval Δx , they distribute throughout the whole space instead, so we should study this new point set, which is also dense point set, for simplicity but lose no generality, we consider finite space such as $x \in [0, 1]$, first, we may divide the whole space in small equal interval Δx , the dividing points are denoted as x_i , then we can define and calculate the measure of the local dense point set in the space interval Δx near each x_i , which can be written as $M_{\Delta x, \Delta t}(x_i, t_j)$, since the measure sum of all local dense point sets in time interval Δt just equals to the length of the continuous time interval Δt , we have:

$$\sum_i M_{\Delta x, \Delta t}(x_i, t_j) = \Delta t \quad (21)$$

On the other hand, since the measure $M_{\Delta x, \Delta t}(x_i, t_j)$ will also turn to be zero when the intervals Δx and Δt turn to be zero, it is not an useful quantity, and we have to create a new quantity on the basis of this measure. Through further analysis, we find that a new quantity $\rho_{\Delta x, \Delta t}(x_i, t_j) = M_{\Delta x, \Delta t}(x_i, t_j) / (\Delta x \cdot \Delta t)$, which can be called average measure density, will be an useful one, it generally does not turn to be zero when Δx and Δt turn to be zero, especially if the limit $\lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \rho_{\Delta x, \Delta t}(x_i, t_j)$ exists, it will no longer relate to the observation sizes Δx and Δt , so it can accurately describe the whole dense point set, as well as all local dense point sets near every instant, now we let:

$$\rho(x, t) = \lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \rho_{\Delta x, \Delta t}(x, t) \quad (22)$$

then we can get:

$$\int_{\Omega} \rho(x, t) dx = 1 \quad (23)$$

this is just the normalization formula, where $\rho(x, t)$ is called position measure density, Ω denotes the whole integral space.

Now, we will analyze the new quantity $\rho(x, t)$ in detail, first, the position measure density $\rho(x, t)$ is not a point quantity, it is defined during infinitesimal interval, this fact is very important, since it means that if the measure density $\rho(x, t)$ exists, then it will be continuous relative to both t and x , that is to say, contrary to the position function $x(t)$, there does not exist the discontinuous situation for the measure density function $\rho(x, t)$, furthermore, this fact also results in that the continuous function $\rho(x, t)$ is the last useful quantity for describing the dense point set; Secondly, the essential meaning of the position measure density $\rho(x, t)$ lies in that it represents the dense degree of the points in the point set in two-dimension space and time, and the points are denser where the position measure density $\rho(x, t)$ is larger.

C. The evolution of dense point set

Now, we will further discuss the evolution law for dense point set.

Just like the continuous position function $x(t)$, although the continuous position measure density function $\rho(x, t)$ completely describes the dense point set, it is one kind of static description about the point set, and it can not be used for prediction itself, so in order to predict the evolution of the dense point set we must create some kind of quantity describing its change, enlightened by the theory of fluid mechanics we can define the fluid density for the position measure density $\rho(x, t)$ as follows:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial j(x, t)}{\partial x} = 0 \quad (24)$$

we call this new quantity $j(x, t)$ position measure fluid density, this equation measure conservation equation, it is evident that this quantity just describes the change of the measure density of dense point set, thus the general evolution equations of dense point set can be written as in the following:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial j(x, t)}{\partial x} = 0 \quad (25)$$

$$\frac{\partial j(x,t)}{\partial t} + \dots = 0 \quad (26)$$

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